
CHAPTER

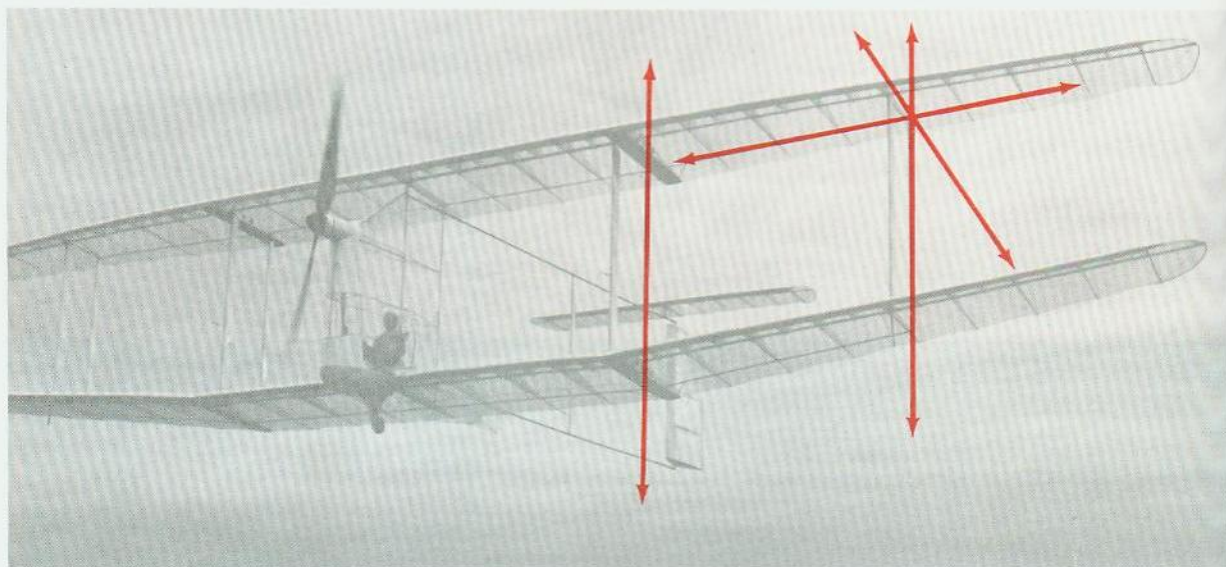
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Parallel Lines and Planes

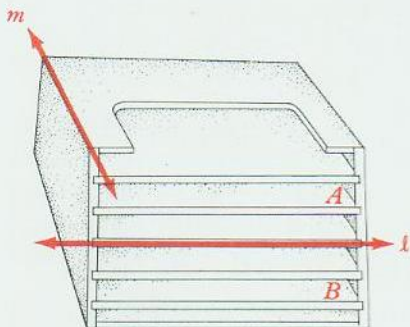


5-1 Basic Definitions



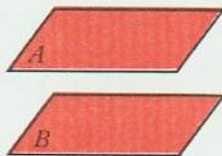
College students built this bi-plane and named it Chrysalis. It is pedal driven, much like a bicycle. During the summer of 1979 the plane made 320 flights.

This bi-plane uses several concepts that will be studied in this lesson, such as parallel lines, lines parallel to a plane, parallel planes, and non-parallel lines that do not intersect.



Lines l and m do not lie in the same plane and they do not intersect. We call them *skew lines*.

Line l is parallel to plane A .



Planes A and B have no points in common. They are *parallel planes*.

Definition 5-1

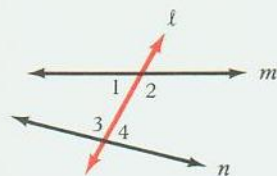
Skew lines are two nonintersecting lines that do not lie in the same plane.

Definition 5-2

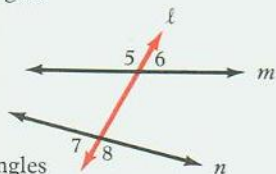
A line and a plane are parallel if they have no points in common.

Definition 5-3

Parallel planes are planes that have no points in common.



Interior angles



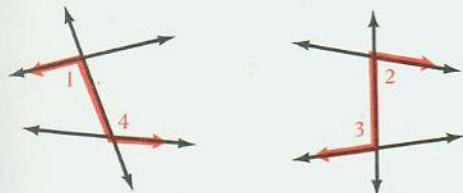
Exterior angles

Line l intersects lines m and n in two different points to form *interior angles* and *exterior angles*. l is called a *transversal*.

Definition 5-4

A **transversal** is a line that intersects two coplanar lines in two different points.

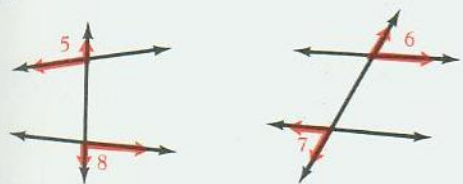
Two lines cut by a transversal form angles that are important in our study of parallel lines.



$\angle 1$ and $\angle 4$ are called *alternate interior angles*.

$\angle 2$ and $\angle 3$ are called *alternate interior angles*.

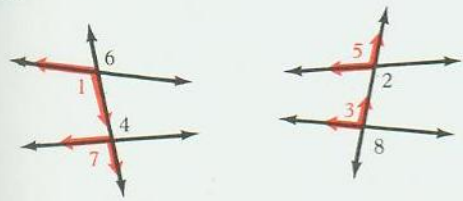
Alternate interior angles are two interior angles with different vertices on opposite sides of the transversal.



$\angle 5$ and $\angle 8$ are called *alternate exterior angles*.

$\angle 6$ and $\angle 7$ are called *alternate exterior angles*.

Alternate exterior angles are two exterior angles with different vertices on opposite sides of the transversal.



There are four pairs of *corresponding angles*: $\angle 1$ and $\angle 7$; $\angle 6$ and $\angle 4$; $\angle 5$ and $\angle 3$; $\angle 2$ and $\angle 8$.

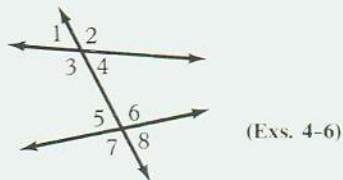
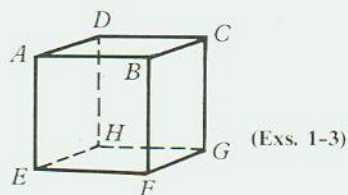
Corresponding angles are on the same side of the transversal. One of the angles is an exterior angle, one is an interior angle.

EXERCISES

A.

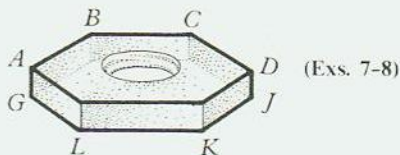
Exercises 1–3 refer to the figure of a cube shown. A model of a cube or a shoe box may help you visualize the cube.

1. Name four lines that are skew with $\overleftrightarrow{AB}$.
2. Name six lines that are parallel to plane $ABCD$.
3. Name three pairs of parallel planes.
4. Name two pairs of alternate interior angles.
5. Name two pairs of alternate exterior angles.
6. Name the angle that corresponds with $\angle 1$.



B.

7. This figure shows a hexagonal-shaped nut. Name three pairs of parallel planes.
8. Several lines containing edges of this nut are skew with $\overleftrightarrow{AB}$. How many?



Activity

How many points of intersection are formed by a given number of lines? It depends upon their positions relative to one another.

Example: Three lines can form:



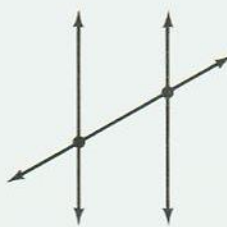
0 points,

or



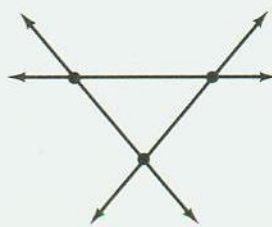
1 point,

or



2 points,

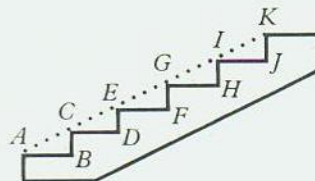
or



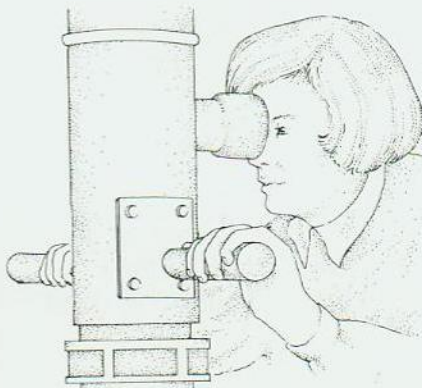
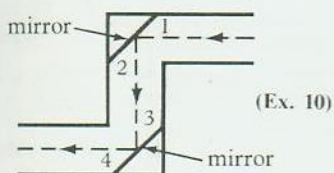
3 points

Experiment! See if four lines can be arranged to form one point of intersection, two points, three points, . . . , more than six points. What about five lines?

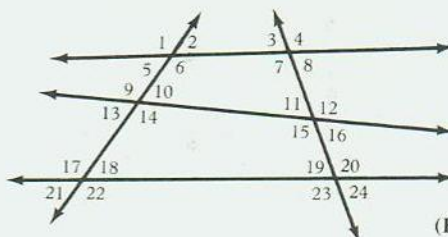
9. A carpenter builds a stairway by cutting triangles like $\triangle ABC$ and $\triangle CDE$ from a piece of lumber. $\angle DCE$ and $\angle FEG$ are corresponding angles relative to what pair of parallel lines and what transversal?



10. In a periscope a pair of mirrors are mounted parallel to each other as shown. The path of light becomes a transversal. Which pair of angles is an alternate interior pair? $\angle 1$ and $\angle 3$, $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$, or $\angle 2$ and $\angle 4$?



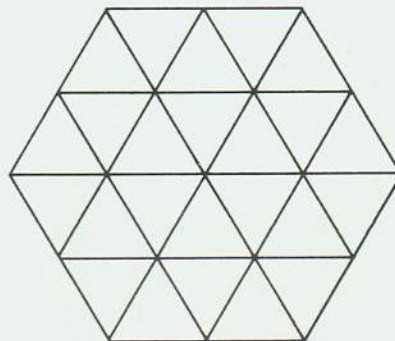
11. Name two pairs of alternate interior angles both of which include $\angle 14$.
12. Name three pairs of alternate exterior angles all of which include $\angle 21$.



PROBLEM SOLVING

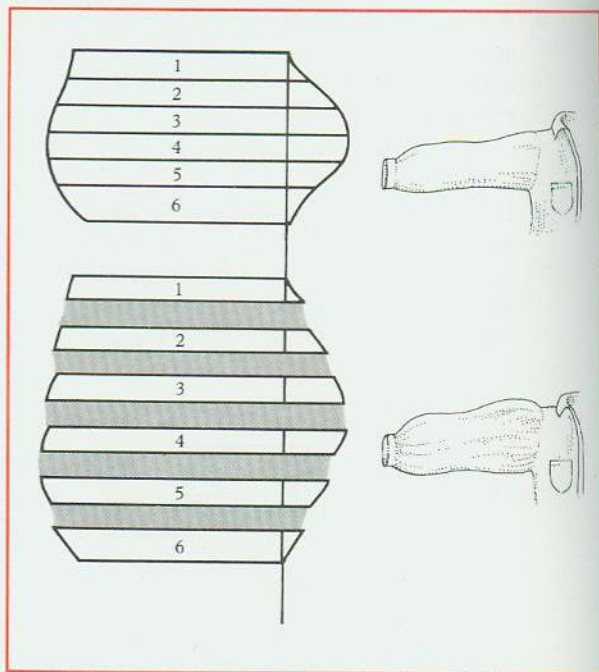
Make a grid of parallel lines like the one on the right.

- Place five red markers on five intersections so that no two are on the same line.
- Then place five blue markers on five intersections so that no two markers are on the same line.

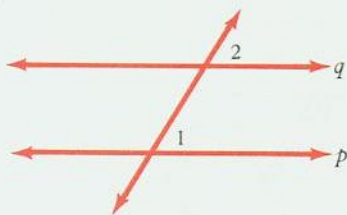


5-2 Theorems About Parallel Lines

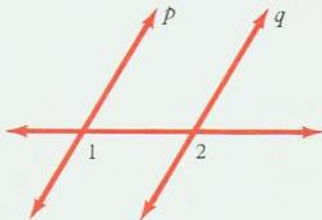
Parallel lines are used every day in a variety of ways. Steel workers install steel beams parallel to one another. Surveyors often determine property lines that are parallel to one another. Clothing designers also use parallel lines. Shown at the right is a basic pattern for a blouse sleeve. To make a pattern for a sleeve that has more fullness, the designer constructs parallel lines on the pattern. Then the strips formed by these parallel lines are cut out and spread apart to form the new pattern.



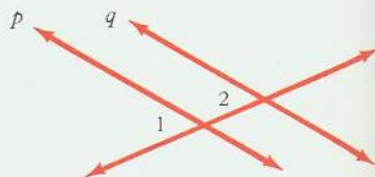
The pairs of angles formed by a pair of lines and a transversal are important in constructing parallel lines. These three figures suggest an important relationship.



Given: $\angle 1 \cong \angle 2$
Observe that $p \parallel q$.



Given: $\angle 1 \cong \angle 2$
Observe that $p \parallel q$.

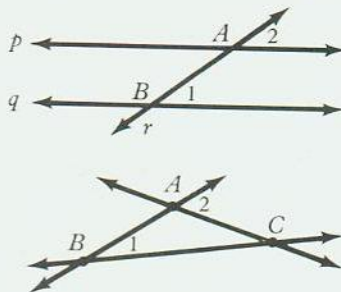


Given: $\angle 1 \cong \angle 2$
Observe that $p \parallel q$.

Theorem 5-1 If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the lines are parallel.

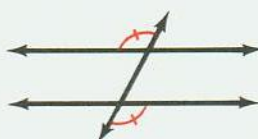
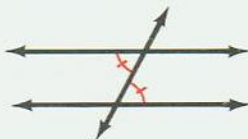
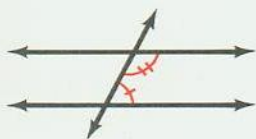
PROOF**Given:** Lines p , q , and r with $\angle 1 \cong \angle 2$ **Prove:** $p \parallel q$

Plan: Assume that $p \nparallel q$. (note: $\nparallel$ means “is not parallel to”) Then consider the triangle that would be formed, and find a contradiction.



Statements	Reasons
1. Suppose $p \nparallel q$.	1. Indirect proof assumption
2. Then p and q intersect at a point, say C , and $\triangle ABC$ is formed.	2. Restatement of 1
3. $\angle 2$ is an exterior angle of $\triangle ABC$.	3. Definition of exterior angle
4. $\angle 1$ is a remote interior angle of $\angle 2$.	4. Definition of remote interior angle
5. $m\angle 2 > m\angle 1$	5. Exterior Angle Theorem
6. $m\angle 1 = m\angle 2$ (contradiction to $m\angle 2 > m\angle 1$)	6. Given
7. Therefore, $p \parallel q$	7. Logic of indirect proof

There are three other related theorems. Match these figures with the theorems.



Theorem 5-2 If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.

Theorem 5-3 If two lines are cut by a transversal and a pair of alternate exterior angles are congruent, then the lines are parallel.

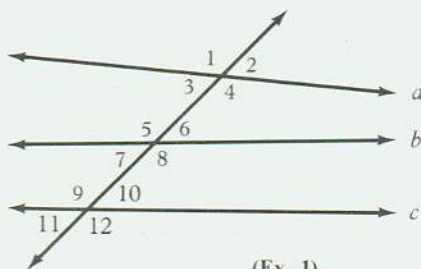
Theorem 5-4 If two lines are cut by a transversal and a pair of interior angles on the same side of the transversal are supplementary, then the lines are parallel.

EXERCISES

A.

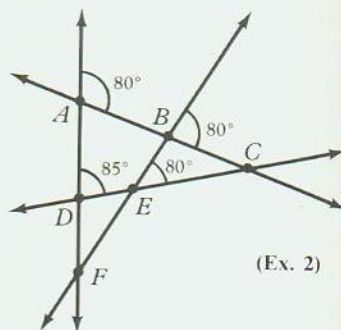
1. In each case below, which lines could you conclude are parallel? Which theorem justifies your answer?

- $\angle 1 \cong \angle 9$
- $\angle 3 \cong \angle 6$
- $m\angle 8 + m\angle 10 = 180$
- $\angle 4 \cong \angle 9$
- $\angle 8 \cong \angle 12$
- $\angle 1 \cong \angle 8$

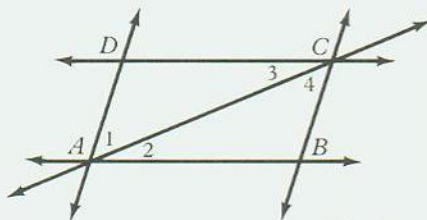


(Ex. 1)

- List all the contradictory information in the figure at the right.
- Name four ways of proving that two lines are parallel.
- Study the photo on page 170. Point out some examples of parallel segments and a transversal.
- What angles could you prove congruent in order to conclude that $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ in the figure?

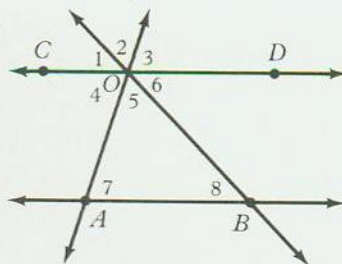


(Ex. 2)



(Ex. 5)

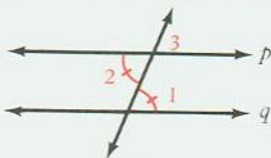
- Which pairs of angles could you prove congruent to show that $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$ in the figure?
- Which pairs of angles could you prove supplementary to conclude that $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB}$ in the figure?



(Exs. 6, 7)

B.

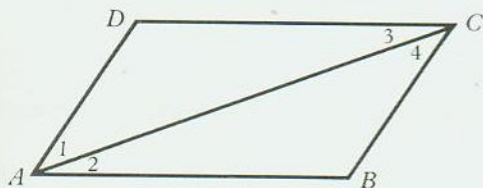
8. Complete the following two-column proof of Theorem 5-2.

Given: $\angle 1 \cong \angle 2$ **Prove:** $p \parallel q$ 

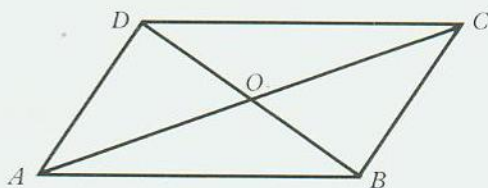
Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. ?
2. $\angle 2 \cong \angle 3$	2. ?
3. $\angle 1 \cong \angle 3$	3. Transitive Property of Congruence
4. $p \parallel q$	4. ?

For exercises 9-12, write a complete two-column proof.

- 9.
- Given:**
- $\overline{AB} \cong \overline{DC}$
-
- $\overline{AD} \cong \overline{BC}$

Prove: $\overline{AB} \parallel \overline{DC}$ (Hint: First prove $\triangle ABC \cong \triangle CDA$.)

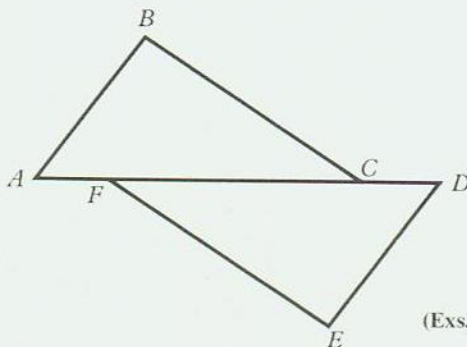
- 10.
- Given:**
- $\overline{DO} \cong \overline{OB}$
-
- $\overline{AO} \cong \overline{OC}$

Prove: $\overline{AB} \parallel \overline{DC}$ 

- 11.
- Given:**
- $\overline{AB} \cong \overline{DE}$
-
- $\overline{BC} \cong \overline{EF}$
-
- $\overline{AF} \cong \overline{CD}$

Prove: $\overline{BC} \parallel \overline{FE}$

- 12.
- Given:**
- $\overline{BC} \cong \overline{EF}$
-
- $\angle BCA \cong \angle EFD$
-
- $\overline{AF} \cong \overline{CD}$

Prove: $\overline{BA} \parallel \overline{DE}$ 

(Exs. 11, 12)

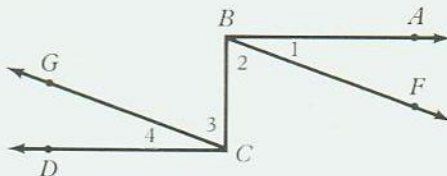
13. Prove that if two lines in a plane are perpendicular to a line, then they are parallel.

14. Prove Theorem 5-3.

15. Prove Theorem 5-4.

C.

16. **Given:** $\overline{AB} \perp \overline{BC}$
 $\overline{DC} \perp \overline{BC}$
 $\angle 1 \cong \angle 4$

Prove: $\overleftrightarrow{BF} \parallel \overleftrightarrow{GC}$ 

(Exs. 16-18)

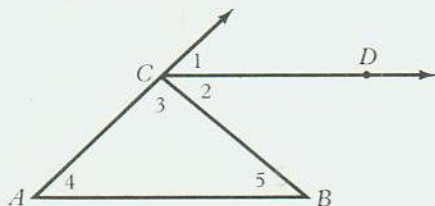
17. **Given:** $\angle ABC \cong \angle BCD$
 $\overrightarrow{BF}$ bisects $\angle ABC$.
 $\overrightarrow{CG}$ bisects $\angle BCD$.

Prove: $\overleftrightarrow{BF} \parallel \overleftrightarrow{CG}$

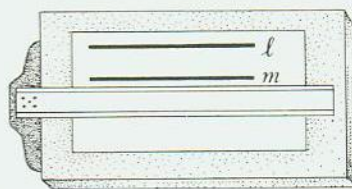
18. **Given:** $\angle 2 \cong \angle 3$
 $\angle 1 \cong \angle 4$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

19. **Given:** $m\angle 2 + m\angle 3 + m\angle 5 = 180$
 $\angle 4 \cong \angle 5$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ 

20. A draftsman uses a T-square to draw a pair of parallel lines across a page. Why can you be sure that these lines are parallel?

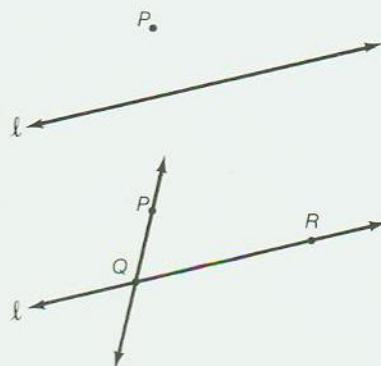


Activity



Draw a line ℓ and a point P not on that line.

Using only a compass and a straight edge construct a line through P that is parallel to ℓ . (Hint: Begin by drawing a line through P that intersects line ℓ .)



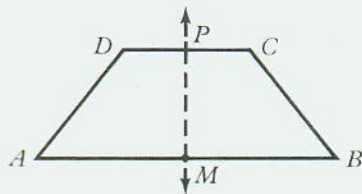
21. **Given:** $\overleftrightarrow{PM}$ is the perpendicular bisector of $\overline{AB}$.

$$\angle A \cong \angle B$$

$$\overline{AD} \cong \overline{BC}$$

Prove: $\overline{AB} \parallel \overline{CD}$

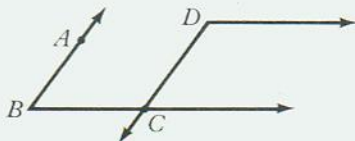
(Hint: Draw some auxiliary lines. An auxiliary line is a line that the solver adds to a figure in order to help prove a theorem or solve a problem.)



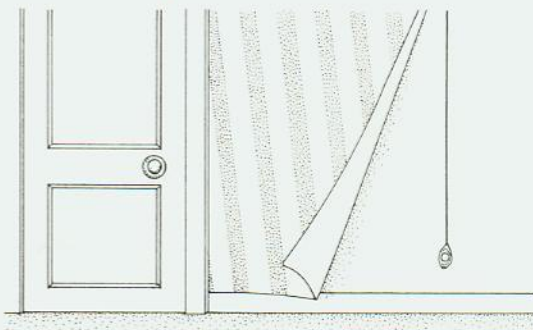
22. **Given:** $\angle BCD \cong \angle D$

$$m\angle B + m\angle D = 180$$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$



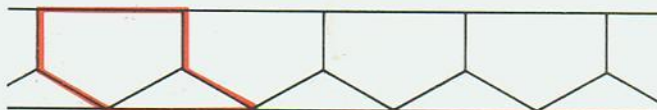
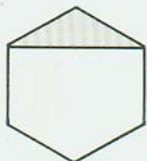
23. A plumb line (a weight hanging by a string) is used to strike a chalk line on a wall. If the first sheet of wallpaper is hung along the chalk line, why should this ensure that the edge of the wallpaper is parallel to the doors, windows, and corners of the room?



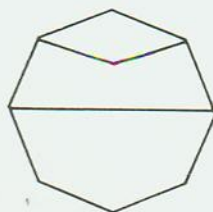
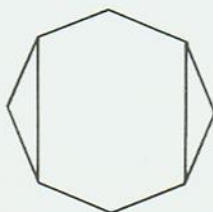
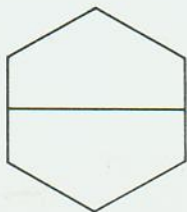
PROBLEM SOLVING

Some polygonal regions can be cut into pieces that can be arranged to form a new polygonal region. Copies of the pieces completely cover the strip between a pair of parallel lines.

Example:



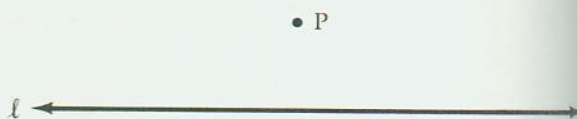
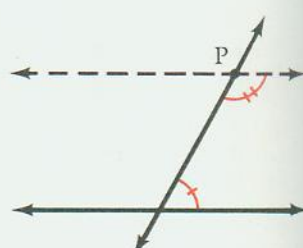
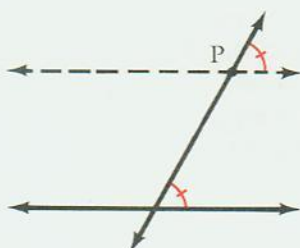
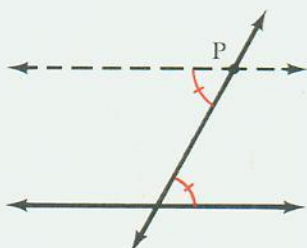
Show that the polygonal regions with the indicated cuts can be arranged to cover such a parallel strip.



5-3 The Parallel Line Postulate

In the last activity you constructed a pair of parallel lines.

There are actually three related methods that could be summarized by these three figures. The correctness of these constructions can be established by using the theorems from the last section.



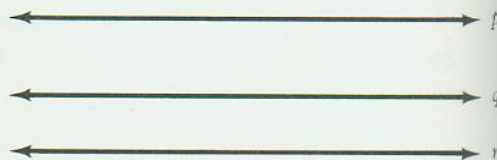
REVIEW: Construct a line through P that is parallel to l .

It seems natural to assume, as we do in the Parallel Postulate, that there is only one line through P that is parallel to l . But why is it true? That question is historically significant (see the next page).

Parallel Postulate

Given a line l and a point P not on l , there exists only one line through P parallel to l .

The proof of this next theorem illustrates the use of the Parallel Postulate.



Theorem 5-5 Given lines p , q , and r , if $p \parallel q$ and $q \parallel r$, then $p \parallel r$.

PROOF

Given: Lines p , q , and r are three distinct lines, $p \parallel q$, $q \parallel r$

Prove: $p \parallel r$



Statements	Reasons
1. Suppose $p \nparallel r$	1. Indirect proof assumption
2. There is a point common to p and r . Call it A .	2. Restatement of 1
3. $p \parallel q$	3. Given
4. $r \parallel q$	4. Given
5. Lines p and r are two distinct lines through A parallel to q . (Contradiction of the Parallel Postulate which asserts that there is only one line through A parallel to q .)	5. Statements 3 and 4
6. Therefore $p \parallel r$	6. Logic of indirect proof



Karl Friedrich Gauss

HISTORICAL SIGNIFICANCE OF THE PARALLEL POSTULATE

For many centuries mathematicians attempted to prove that the Parallel Postulate was a theorem. Repeatedly these attempts failed. In the early nineteenth century three mathematicians, Karl Friedrich Gauss (1777–1855), Janos Bolyai (1802–1860), and Nicolai Ivanovitch Lobachevsky (1793–1856), working independently of one another, tried to eliminate the Parallel Postulate from the Euclidean Postulate System and prove it as a theorem. They used an indirect method. But rather than arriving at a contradiction, they found that this assumption led to an entirely new set of theorems—a totally new geometry. This important mathematical discovery led to what is called non-Euclidean geometry.

EXERCISES

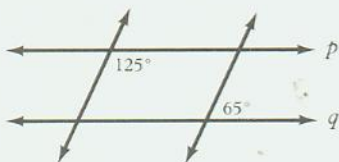
A.

1. State the Parallel Postulate in your own words.
2. Which two words in the Parallel Postulate are the most important?
3. Which of these statements are true and which are false?
 - a. There is a line through A that is parallel to ℓ .
 - b. We could prove that there is a line through A parallel to ℓ even without using the Parallel Postulate.
 - c. The Parallel Postulate says that there is only one line through A parallel to ℓ .
 - d. If p is a line through A perpendicular to ℓ and q is a line through A perpendicular to p , then $q \parallel \ell$.

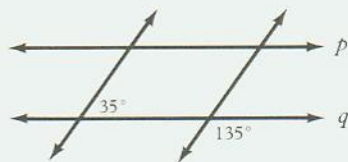
• A 

In exercises 4 and 5 determine whether the lines p and q can be proved parallel.

4.



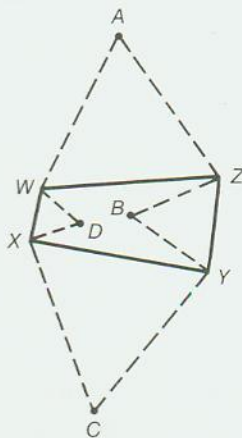
5.



Activity



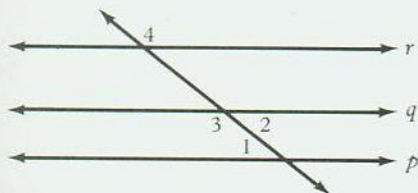
1. Draw with a straightedge any quadrilateral $WXYZ$.
2. Construct equilateral triangles on each side of the quadrilateral alternately inside and outside the quadrilateral and label the new vertices A, B, C, D .
3. What is true about $\vec{AD}$ and $\vec{BC}$? About $\vec{AB}$ and $\vec{CD}$?
4. State a generalization.



B.

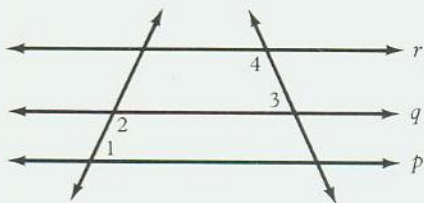
6. **Given:** $\angle 1 \cong \angle 2$
 $\angle 3 \cong \angle 4$

Prove: $p \parallel r$



7. **Given:** $m\angle 1 + m\angle 2 = 180$
 $m\angle 3 + m\angle 4 = 180$

Prove: $p \parallel r$



8. Given line ℓ and P , construct the perpendicular from P to ℓ . Call it m . Construct the line $\perp$ to m through P . Call this line r . What is the relationship between r and ℓ ? Why?

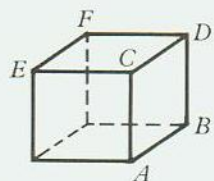
9. Show that the Activity on p. 178 results in a pair of parallel lines.



C.

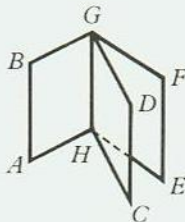
For these exercises you may assume that Theorem 5-5 is true for lines in space in addition to lines in a plane.

10. If $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are the edges of a cube as shown, show that $\overrightarrow{AB} \parallel \overrightarrow{EF}$.



(Ex. 10)

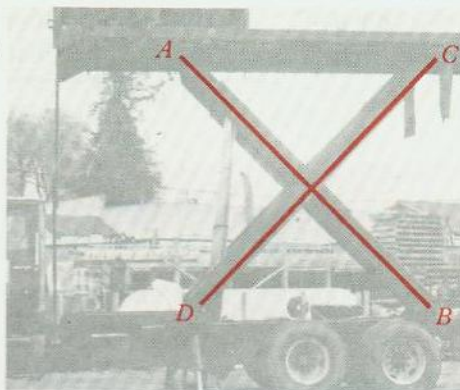
11. If $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ are edges of three pages of a book (assume rectangular pages), prove that $\overrightarrow{AB}$, $\overrightarrow{CD}$, and $\overrightarrow{EF}$ are parallel to each other.



PROBLEM SOLVING

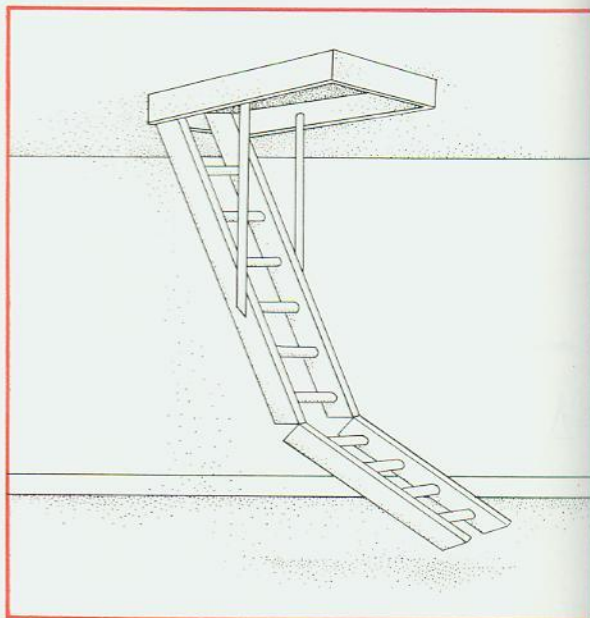
This photograph shows a flat-bed scissors truck raised to its maximum height. The support pieces labeled $\overline{AB}$ and $\overline{CD}$ bisect each other.

Explain why these conditions mean that the bed of the truck is parallel to the truck frame.

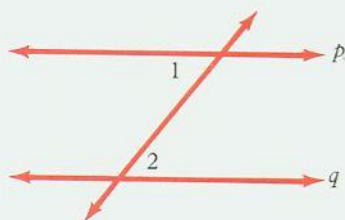


5-4 More Theorems About Parallel Lines

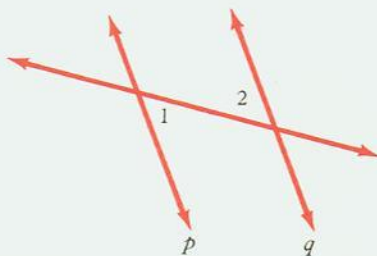
Many “do-it-yourselfers” purchase a pre-assembled fold-down ladder to install in the ceiling of the house or garage. These ladders are all manufactured the same length. The installer must determine how to cut the bottom section so that the ladder will sit firmly on the floor. Both the length and the angle must be considered. The fact that the ceiling and floor are parallel is important to the solution of this problem.



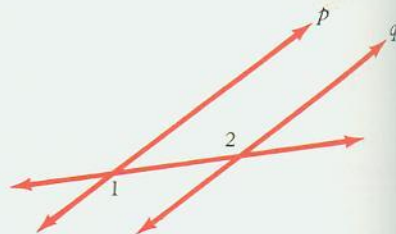
In these figures how are $\angle 1$ and $\angle 2$ related?



Given: $p \parallel q$
Observe that $\angle 1 \cong \angle 2$.



Given: $p \parallel q$
Observe that $\angle 1 \cong \angle 2$.



Given: $p \parallel q$
Observe that $\angle 1 \cong \angle 2$.

These figures suggest the following theorem.

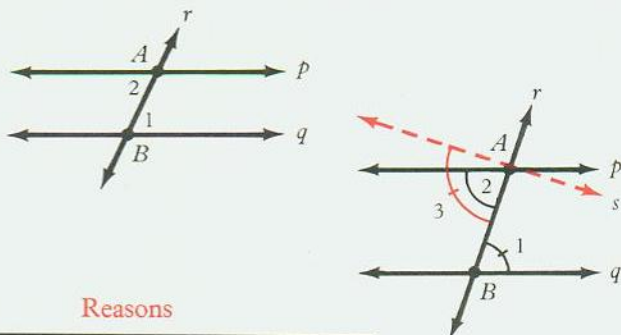
Theorem 5-6 If two parallel lines are cut by a transversal, then alternate interior angles are congruent.

PROOF

Given: Lines $p \parallel q$ with transversal r .
 $\angle 1$ and $\angle 2$ are alternate interior angles.

Prove: $\angle 1 \cong \angle 2$

Plan: Assume that $\angle 1$ is not congruent to $\angle 2$, and find a contradiction.



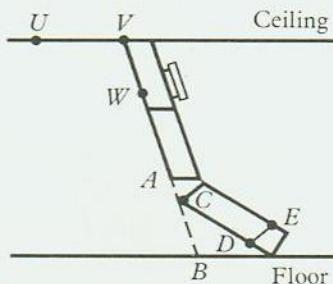
Statements	Reasons
1. $\angle 1$ is not congruent to $\angle 2$.	1. Indirect proof assumption
2. Construct a line through A so that $\angle 1 \cong \angle 3$, and $\angle 1$ and $\angle 3$ are alternate interior angles.	2. Construction
3. $q \parallel s$, A is on s .	3. If alternate interior angles are congruent, then the lines are parallel.
4. $p \parallel q$, A is on p .	4. Given
5. There are two lines through A parallel to q . (Contradiction of the Parallel Postulate)	5. Statements 3 and 4
6. $\angle 1 \cong \angle 2$	6. Logic of indirect proof

APPLICATION

Theorem 5-6 can be used to solve the problem raised at the beginning of the lesson. How should the bottom sections be cut? Follow this procedure.

1. Measure AB and locate point D so that $AB = CD$.
2. Measure $\angle UVW$ and locate point E so that $m\angle UVW = m\angle CDE$.

There are several additional theorems stated here.



Theorem 5-7 If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.

Theorem 5-8 If two parallel lines are cut by a transversal, then corresponding angles are congruent.

Theorem 5-9 If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.

EXERCISES

A.

In the figure for exercises 1-7 lines p and q are parallel and $m\angle 3 = 55$.

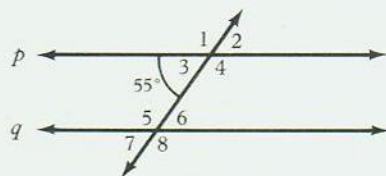
1. $m\angle 1 = ?$

2. $m\angle 2 = ?$

3. $m\angle 4 = ?$

5. $m\angle 6 = ?$

6. $m\angle 7 = ?$



(Exs. 1-7)

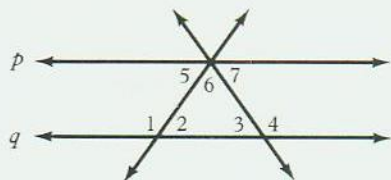
In the figure for exercises 8-12, lines p and q are parallel, $m\angle 1 = 125$ and $m\angle 4 = 143$.

8. $m\angle 2 = ?$

9. $m\angle 3 = ?$

10. $m\angle 5 = ?$

11. $m\angle 7 = ?$



(Exs. 8-12)

In the figure for exercises 13-22, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$. Also $m\angle ADC = 110$ and $m\angle ACD = 28$.

13. $m\angle 1 = ?$

14. $m\angle 10 = ?$

15. $m\angle 3 = ?$

16. $m\angle 4 = ?$

17. $m\angle 5 = ?$

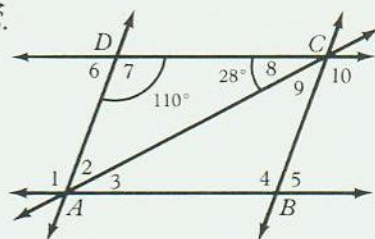
18. $m\angle 6 = ?$

19. $m\angle BCD = ?$

20. $m\angle 9 = ?$

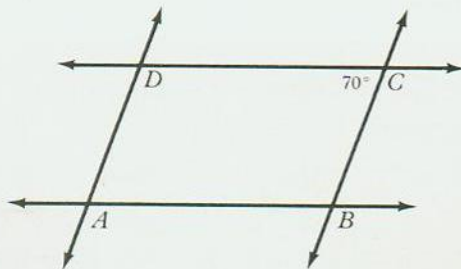
21. $m\angle 2 = ?$

22. $m\angle BAD = ?$



(Exs. 13-22)

23. Assume that $m\angle BCD = 70$ in the figure. What must the measures of $\angle ABC$, $\angle CDA$, and $\angle DAB$ be if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.



B.

For exercises 24 and 25, assume that lines p and q are parallel.

24. If $m\angle 1 = 2x + 3$ and $m\angle 4 = 7x - 12$, find $m\angle 1$ and $m\angle 2$.

25. If $m\angle 1 = 2x + 4$, $m\angle 5 = 3y + 6$, and $m\angle 2 = 4y + 6$, find the measures of $\angle 1$, $\angle 2$, and $\angle 5$.

26. Prove Theorem 5-8.

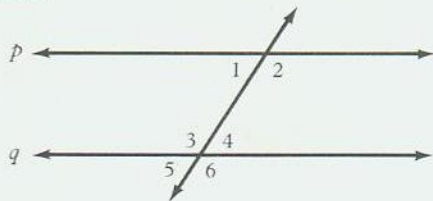
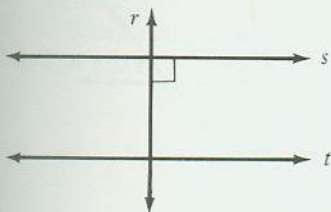
27. Prove Theorem 5-9.

28. Prove Theorem 5-7.

29. **Given:** $s \parallel t$

$$r \perp s$$

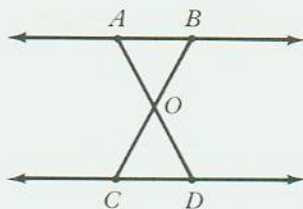
Prove: $r \perp t$



(Exs. 24, 25)

30. **Given:** $\overline{AO} \cong \overline{OD}$
 $\overline{BO} \cong \overline{OC}$

Prove: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



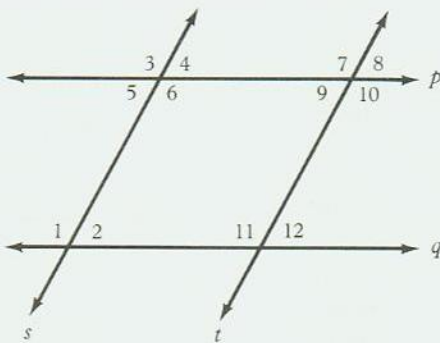
31. **Given:** $p \parallel q$

$$s \parallel t$$

Prove: $\angle 1 \cong \angle 7$

$$\angle 2 \cong \angle 9$$

(Exs. 31-33)



32. **Given:** $p \parallel q$

$$\angle 3 \cong \angle 11$$

Prove: $s \parallel t$

33. **Given:** $s \parallel t$

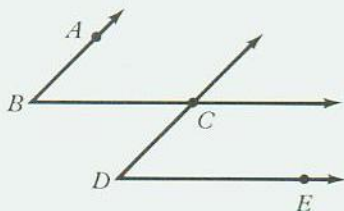
$\angle 9$ and $\angle 1$ are supplementary

Prove: $p \parallel q$

C.

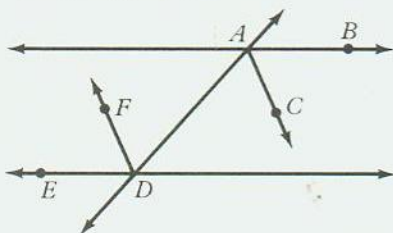
34. Given: $\overline{AB} \parallel \overline{CD}$
 $\overline{BC} \parallel \overline{DE}$

Prove: $\angle B \cong \angle D$



36. Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{DE}$
 $\overleftrightarrow{AC}$ bisects $\angle BAD$.
 $\overleftrightarrow{DF}$ bisects $\angle ADE$.

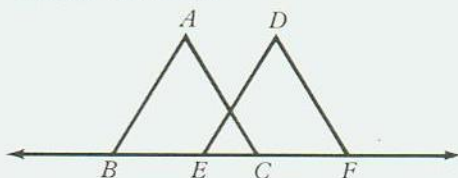
Prove: $\overleftrightarrow{AC} \parallel \overleftrightarrow{FD}$



35. Given: $\angle A \cong \angle D$
 $\overline{AB} \parallel \overline{DE}$

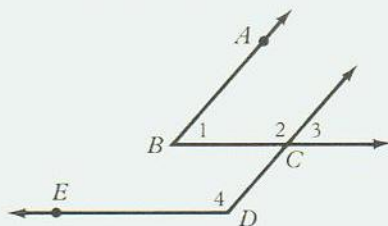
$\overline{AB} \cong \overline{DE}$

Prove: $\overline{BC} \cong \overline{EF}$



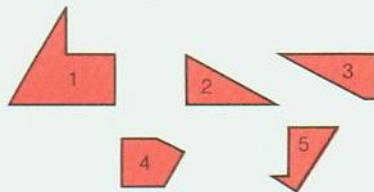
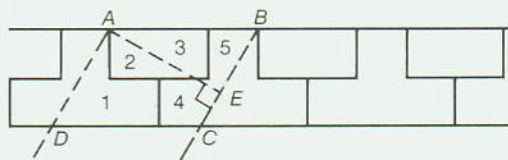
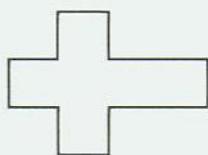
37. Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
 $\overleftrightarrow{BC} \parallel \overleftrightarrow{DE}$

Prove: $m\angle 1 + m\angle 4 = 180$

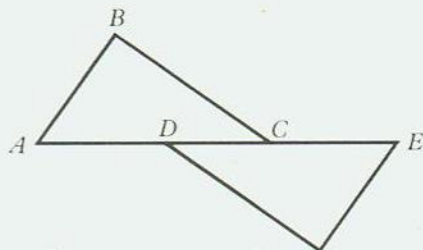


Activity

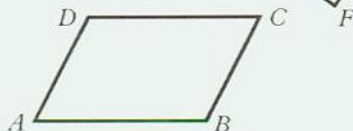
1. Begin with a polygon that can be cut into pieces and rearranged into a repeating strip pattern (as on page 179).
2. Label points A and B to be any two points that correspond to each other in neighboring pieces of the strip.
3. Draw a pair of parallel lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ as shown. Draw $\overline{AE} \perp \overline{BC}$. The pieces 1-5 so obtained can be arranged to form either the original polygon or a rectangle.
4. Repeat this three-step procedure for the polygons in the Problem Solving box on page 179. In each case arrange the pieces you produce to form a rectangle.



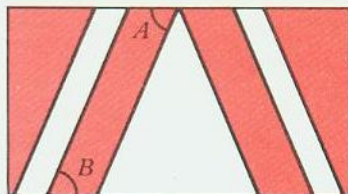
38. **Given:** $\overline{AD} \cong \overline{EC}$
 $\overline{BC} \cong \overline{FD}$
 $\overline{BC} \parallel \overline{FD}$
Prove: $\overline{AB} \parallel \overline{EF}$



39. If $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$,
 prove that $\angle B \cong \angle D$.



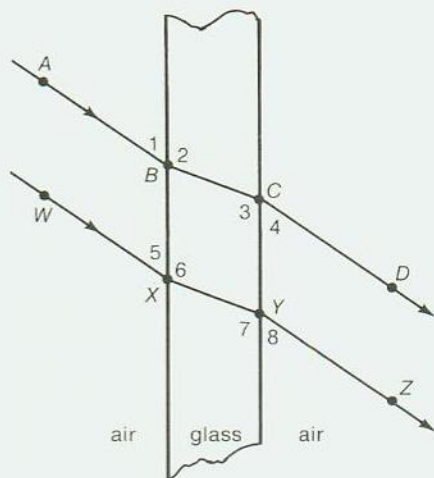
40. **Prove:** If two opposite sides of a quadrilateral are parallel and congruent, then the other two opposite sides are parallel and congruent.
41. An interior decorator is pasting strips of wallpaper on a wall to make a bold stripe pattern.
 When cutting the paper, how does the decorator know that $\angle A$ and $\angle B$ should be congruent?



PROBLEM SOLVING

Light rays are bent as they pass through glass. Assume that a ray is bent the same amount entering a piece of glass as exiting the glass. Explain two things.

- Why does a ray exit in a direction parallel to the direction it entered? That is, why is $\overleftrightarrow{AB}$ parallel to $\overleftrightarrow{CD}$?
- Why do entering parallel rays exit as parallel rays? That is, why does $\overleftrightarrow{AB} \parallel \overleftrightarrow{WX}$ imply $\overleftrightarrow{CD} \parallel \overleftrightarrow{YZ}$?



Important Ideas—Chapter 5

Terms

- Skew lines (p. 170)
- Parallel line and plane (p. 170)
- Parallel planes (p. 170)
- Transversal (p. 171)
- Alternate interior angles (p. 171)
- Alternate exterior angles (p. 171)
- Corresponding angles (p. 171)

Postulate

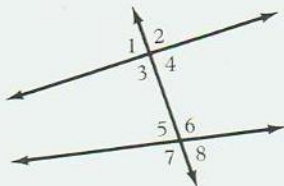
Parallel Postulate: Given a line ℓ and a point P not on ℓ , there exists only one line through P parallel to ℓ .

Theorems

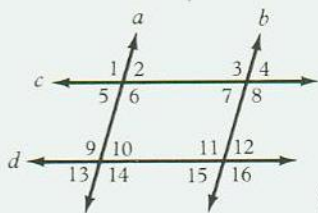
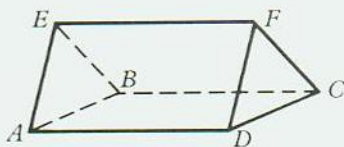
- 5-1 If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the lines are parallel.
- 5-2 If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the lines are parallel.
- 5-3 If two lines are cut by a transversal and a pair of alternate exterior angles are congruent, then the lines are parallel.
- 5-4 If two lines are cut by a transversal and a pair of interior angles on the same side of the transversal are supplementary, then the lines are parallel.
- 5-5 Given lines p , q , and r , if $p \parallel q$ and $q \parallel r$ then $p \parallel r$.
- 5-6 If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
- 5-7 If two parallel lines are cut by a transversal, then alternate exterior angles are congruent.
- 5-8 If two parallel lines are cut by a transversal, then corresponding angles are congruent.
- 5-9 If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.

Chapter 5—Review

- Name pairs of alternate interior angles.
 - Name pairs of alternate exterior angles.
 - Name pairs of corresponding angles.
- Indicate on your paper whether the following statements are true or false.
 - If two lines are parallel and are cut by a transversal, then the corresponding angles are congruent.
 - If two lines are cut by a transversal to form alternate interior angles, then the lines are parallel.
 - If two lines are parallel to a third line, then the two lines are parallel.

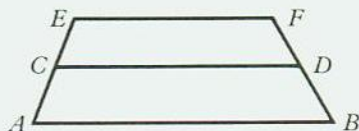


- Name a pair of parallel planes.
 - Name a pair of skew lines.
- If $m\angle 6 = 120$ and $m\angle 12 = 60$, can you prove $a \parallel b$?
- If $m\angle 13 = 55$, $a \parallel b$, and $c \parallel d$, find $m\angle 4$.
- Given:** $a \parallel b$, $c \parallel d$, $m\angle 1 = 8x - 2$, $m\angle 11 = 7x + 11$.
Solve for x and find $m\angle 10$.
- Given:** $c \parallel d$
 $\angle 8$ and $\angle 10$ are supplementary



(Exs. 4-7)

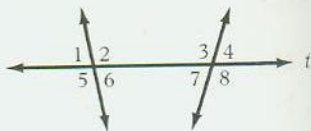
- Prove:** $a \parallel b$
- 8. Given:** $\angle ECD \cong \angle EAB$
Prove: $\angle FDC \cong \angle FBA$



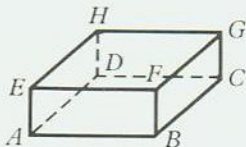
(Ex. 8)

Chapter 5—Test

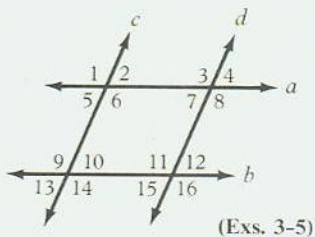
1. a. Name a pair of alternate interior angles that includes $\angle 2$.
b. Name a pair of corresponding angles that includes $\angle 2$.



2. a. Name a pair of parallel planes.
b. Name a pair of skew lines.



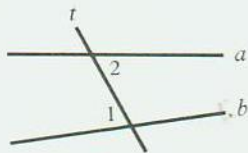
3. If $a \parallel b$, $m\angle 10 = 70$, and $m\angle 7 = 70$, is $c \parallel d$?
4. If $m\angle 6 = 115$, $m\angle 7 = 65$ and $m\angle 15 = 65$, find $m\angle 13$.



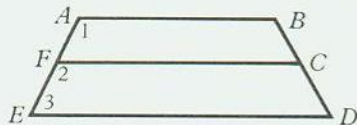
(Exs. 3-5)

5. **Given:** $a \parallel b$, $c \parallel d$, $m\angle 11 = 7x$ and $m\angle 8 = 5x + 32$.
Solve for x and find $m\angle 6$.

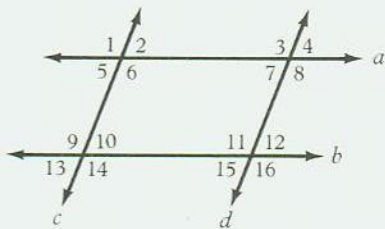
6. If a intersects b , can $m\angle 1 = m\angle 2$? Explain.



7. **Given:** $\angle 1 \cong \angle 2$
 $\angle 2$ and $\angle 3$ are supplementary
Prove: $\overline{AB} \parallel \overline{ED}$



8. **Given:** $c \parallel d$, $\angle 8 \cong \angle 14$
Prove: $a \parallel b$



Algebra Review

Solve each equation for x .

1. $3x + 5 = 21 - x$

2. $10 - 6x = 4 + 2(1 - x)$

3. $5 - x = -3x$

4. $5 + 2(x - 4) = 19$

5. $\frac{1}{2}(2x - 6) = 2x + 5$

6. $\frac{2}{3}x + \frac{3}{4} = 1$

7. $4(x - 2) = 4 - (x + 2)$

8. $\frac{1}{2}x + 2(x - 4) = 17$

9. $2(x + 4) = 3(4 - x)$

Solve each inequality for x .

10. $-x > 9$

11. $x + 2 < 7$

12. $2x + 6 > 17$

13. $3(x - 2) < 8$

14. $15 - 2x < x$

15. $\frac{2x}{3} > 24$

16. $3x + 7 > 2x - 4$

17. $2|x| < 10$

18. $|x + 2| > 14$

Evaluate. Express fractions in lowest terms.

19. $\sqrt{8}$

20. $\sqrt{640}$

21. $\sqrt{27} + \sqrt{3}$

22. $8(8 - 6\sqrt{16})$

23. $\sqrt{75} - \sqrt{12}$

24. $\sqrt{108} + 3\sqrt{3}$

25. $\frac{7\sqrt{196}}{14\sqrt{64}}$

26. $\sqrt{\frac{4}{9}}$

27. $\sqrt{18} - \frac{\sqrt{8}}{8}$

Solve each system of equations for x and y .

28. $x = 2$

29. $2x + y = 14$

30. $x = 2y + 1$

$x + y = 5$

$3x + y = 4$

$y = 2x - 20$

31. $x - y = 2$

32. $3x + 4y = 14$

33. $x + y = 10$

$x + y = 6$

$x - 3y = -17$

$x - y = 18$

34. $3x + 4y = 18$

35. $y = 2x$

36. $2x + y = 6$

$6x - 4y = 0$

$x + 3y = 49$

$-3x + 2y = 8\frac{1}{2}$

Solve.

37. A rectangle has a width three times its length. Find the length and width of the rectangle if its perimeter is 24.

38. A linear pair of angles have measures of $x + 20$ and $x + 30$. Find the measure of each angle.

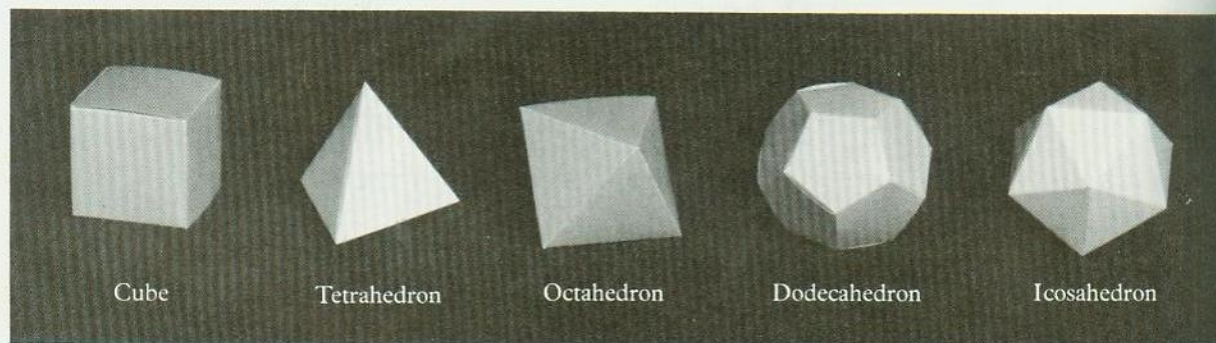
Geometry In Our World

Mineralogy: Symmetry

Minerals, as a rule, have regular arrangements of atoms that have some geometrical symmetry. While crystals do not always exhibit perfect symmetry, some have shapes that are close to the five regular polyhedra pictured below. Such crystals have axes of rotational symmetry and planes of reflectional symmetry.



Fluorite



Cube

Tetrahedron

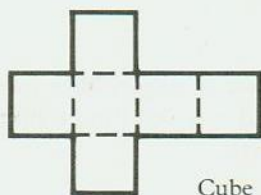
Octahedron

Dodecahedron

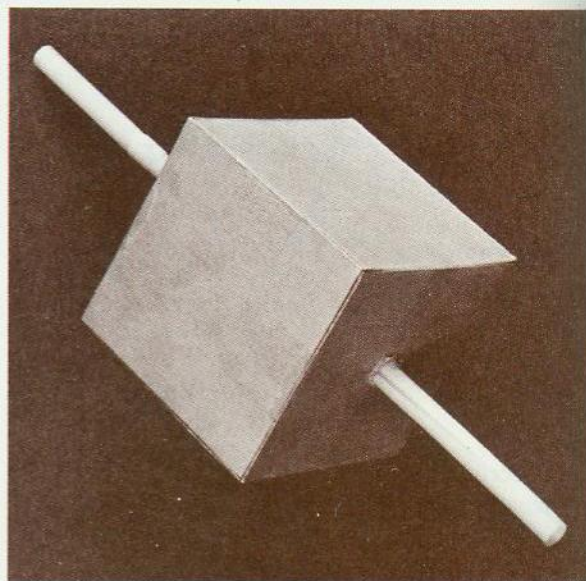
Icosahedron

1 Find axes of rotational symmetry for a solid figure.

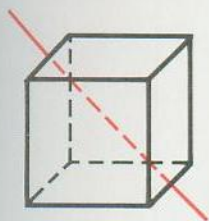
You can fold a pattern as shown here and make a model for a cube. Cut small slots and push a straw through the cube as shown in the picture. If the cube is rotated 90° four times around the straw axis, it returns to the starting position. Since this is true, we say the straw represents an *axis of rotational symmetry of order 4*.



Cube

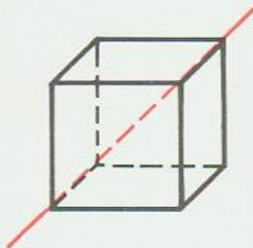


The three types of axes of symmetry for a cube are shown here.



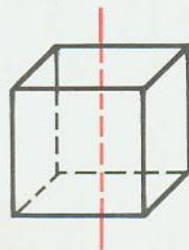
Order 2

A line through the midpoints of opposite edges



Order 3

A line through a pair of opposite vertices



Order 4

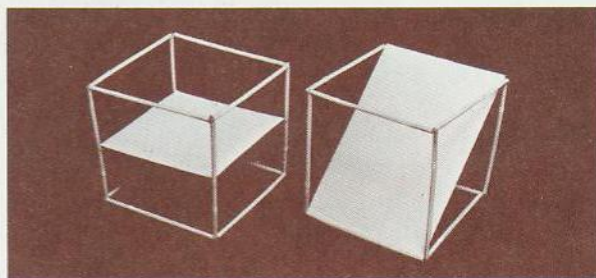
A line through the centers of opposite faces

- How many different axes of symmetry of order 2 are there for a given cube?
- How many different axes of symmetry of order 3 are there for a given cube?
- How many different axes of symmetry of order 4 are there for a given cube?

2

Find planes of reflectional symmetry for a solid figure.

You can construct a model of a cube using 12 congruent drinking straws fastened with pipe cleaners as shown in the picture. Cut "planes" from file cards or posterboard and use them to help you visualize planes of symmetry as shown.



- A plane passing through opposite edges of a cube as shown here is a plane of symmetry for the cube. How many planes of symmetry of this type are there for a given cube?
- A plane midway between a pair of opposite faces of a cube is a plane of symmetry for the cube. How many planes of symmetry of this type are there for a given cube?

